

The table shows values of $f(x)$ and $f'(x)$ at various input values. If $g(x) = \frac{f(x^2)}{x}$, find $g'(3)$.

SCORE: ____ / 4 PTS

x	0	1	2	3	4	5	6	7	8	9
$f(x)$	-4	-1	0	2	1	4	6	5	3	-3
$f'(x)$	3	2	4	0	5	1	-1	-3	-7	-6

$$g'(x) = \frac{f'(x^2) \cdot 2x \cdot x - f(x^2)}{x^2} \quad \textcircled{2\frac{1}{2}}$$

THIS PRODUCT CAN BE WRITTEN DIFFERENTLY, AS LONG AS IT EQUALS -108

$$g'(3) = \frac{f'(9) \cdot 6 \cdot 3 - f(9)}{9} = \frac{(-6)18 - (-3)}{9} = \frac{-35}{3}$$

IF YOUR FINAL ANSWER IS CORRECT, GIVE $\rightarrow$ $\textcircled{1}$ $\textcircled{\frac{1}{2}}$
YOURSELF THIS POINT AUTOMATICALLY

If $f(x) = (\sec x)^{x^3}$, find $f'(x)$.

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$$y = (\sec x)^{x^3}$$

$$\ln y = x^3 \ln \sec x \quad \left(\frac{1}{2}\right)$$

$$\textcircled{1} \frac{1}{y} \frac{dy}{dx} = 3x^2 \ln \sec x + x^3 \frac{1}{\sec x} \sec x \tan x \quad \left(2\frac{1}{2}\right)$$

$$\frac{dy}{dx} = (\sec x)^{x^3} (3x^2 \ln \sec x + x^3 \tan x) \quad \textcircled{1}$$

$$= x^2 (\sec x)^{x^3} (3 \ln \sec x + x \tan x)$$

Prove the derivative of $\cos^{-1} x$. Show all steps and justifications.

SCORE: ____ / 4 PTS

$$\begin{aligned} y &= \cos^{-1} x \longrightarrow y \in [0, \pi] \longrightarrow \sin y \geq 0 \quad (1) \\ \cos y &= x \quad (1) & \text{so } \sin y &= \sqrt{1 - \cos^2 y} \quad (1/2) \\ & & &= \sqrt{1 - x^2} \\ -\sin y \frac{dy}{dx} &= 0 \quad (1/2) \\ \frac{dy}{dx} &= -\frac{1}{\sin y} \quad (1/2) = -\frac{1}{\sqrt{1 - x^2}} \quad (1/2) \end{aligned}$$

If the position function of an object is given by $f(t) = \tan^{-1} t^4$, find the acceleration function.

SCORE: ____ / 5 PTS

$$f'(t) = \frac{1}{1+(t^4)^2} 4t^3 = \boxed{\frac{4t^3}{1+t^8}} \textcircled{2}$$

$$f''(t) = \boxed{\frac{12t^2(1+t^8) - 4t^3(8t^7)}{(1+t^8)^2}} \textcircled{2}$$

$$= \frac{12t^2 + 12t^{10} - 32t^{10}}{(1+t^8)^2}$$

$$= \boxed{\frac{12t^2 - 20t^{10}}{(1+t^8)^2}} \textcircled{1} = \frac{4t^2(3-5t^8)}{(1+t^8)^2}$$

Find $\frac{dy}{dx}$ if $\cos \frac{y}{x} = x^2 y^3$.

SCORE: ____ / 6 PTS

$$\boxed{\left(-\sin \frac{y}{x}\right) \left(\frac{y'x - y}{x^2}\right) = 2xy^3 + 3x^2y^2y'}$$

$$\left(-\sin \frac{y}{x}\right)(y'x - y) = 2x^3y^3 + 3x^4y^2y'$$

$$(-x \sin \frac{y}{x})y' + y \sin \frac{y}{x} = 2x^3y^3 + 3x^4y^2y'$$

$$\boxed{y \sin \frac{y}{x} - 2x^3y^3 = 3x^4y^2y' + (x \sin \frac{y}{x})y'}$$

$$\boxed{= (3x^4y^2 + x \sin \frac{y}{x})y'}$$

$$\frac{dy}{dx} = \boxed{\frac{y \sin \frac{y}{x} - 2x^3y^3}{3x^4y^2 + x \sin \frac{y}{x}}}$$

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EQUATION
FIRST

If $f(x) = \ln(x + \sqrt{x^2 - 1})$, find $f'(x)$. **NOTE: In the final simplified answer, x appears only once.**

SCORE: ____ / 5 PTS

$$f'(x) = \frac{1}{x + \sqrt{x^2 - 1}} \cdot \left(1 + \frac{1}{2\sqrt{x^2 - 1}} \cdot 2x\right)$$

$$= \textcircled{1} \left[\frac{1}{x + \sqrt{x^2 - 1}} \cdot \left(1 + \frac{x}{\sqrt{x^2 - 1}}\right) \right] \textcircled{2}$$

$$= \frac{1}{x + \sqrt{x^2 - 1}} \cdot \left[\frac{\sqrt{x^2 - 1} + x}{\sqrt{x^2 - 1}} \right] \textcircled{1}$$

$$= \left[\frac{1}{\sqrt{x^2 - 1}} \right] \textcircled{1}$$